FACTSET 〉 SEE THE ADVANTAGE

www.factset.com

BOND FUTURES

By Figo Liu

FACTSET 〉 SEE THE ADVANTAGE

BOND FUTURES

Figo Liu
filiu@factset.com
December 2021

Bond Futures

Contents

1 Introduction

Bond Futures are exchange-traded instruments that have an underlying basket of bonds. The contracts are generally executed on government bonds and are commonly referred to as Treasury Bond Futures in the U.S. The basket is selected based on rules about remaining maturity, original maturity, and issue size published by the trading exchange. These are highly liquid instruments used for hedging, directional trading, and arbitrage.

2 Symbology

2.1 Decoding the Future Ticker

The bond future ticker at FactSet has a convention that is typically consistent with exchange rules. It is a combination of the future root, which represents the future underlying, expiration code (month code + year unit), and exchange. For example, the future ticker TYZ19-CBT represents a 10-year U.S. treasury future expiring in December 2019 and trading on CBOT.

3 Quoting Convention

3.1 Price Forward

Most of the bond futures are quoted on forward price basis where the settlement price indicates the cash received by the short party for delivery of the underlying bond.

3.2 Yield

The future price is determined by pricing a standardized bond using quote as yield to maturity. Standard bonds have terms published by the exchange. Swedish bond futures follow this quote convention at FactSet.

3.3 100 - Yield

Similar to Yield convention, the future price is determined by pricing a standardized bond but by using 100 - quote as yield to maturity. Australian bond futures follow this quote convention at FactSet.

4 Settlement Convention

4.1 Physical Delivery

The short party has to deliver the bonds selected from the underlying basket. The cash received by the short is determined from the Conversion Factor calculated based on exchange published rules. This is the primary choice of settlement for a majority of currencies.

4.2 Cash Settled

The settlement price is determined based on quote convention and the contracts are settled in cash without a need for physical bond delivery. Currencies like AUD and KRW have published underlying baskets of bonds but they are only used to determine the settlement quote.

5 Definitions

Cheapest to Deliver (CTD): CTD bond is the one that is most profitable ("cheapest") to deliver for the short position of all available bonds. The short position receives,

(Settlement Price ∗ Conversion Factor) + Accrued Interest

And the cost of purchasing a bond to deliver is,

Bond Price + Accrued Interest

The Cheapest to Deliver bond is the one for which,

Bond Price − (Settlement Price ∗ CF)

is minimum. We determine this based on Implied Repo Rate Calculation.

Factor Rate: The rate stipulated by the exchange specific to each future contract type to be used in conversion factor calculation. For example, all U.S. Treasury futures have a factor rate of six.

Conversion Factor (CF): The conversion factor for a bond is set equal to the quoted price bond would have per dollar of principal assuming that the yield to maturity equals Factor Rate

Implied Repo Rate (IRR): The implied repo rate is the return received by going long basis. This involves buying the cash bond, financing it at the current borrowing or repo rate to term, and then delivering those bonds to satisfy the short futures obligation. Therefore, the higher the implied repo rate, the cheaper the bond is to deliver.

$$IRR = \frac{(FuturesQuote * CF + A_d) - (BondPrice + A_s) + IC}{(BondPrice + A_s) * AF - \sum IC_i * AF_i}$$

Where

Ad = Accrued interest on delivery date

As = Accrued interest on settlement date

IC = Interim coupons between settlement and delivery

AF = Year fraction to delivery date

Cost of Carry: In order to guarantee the delivery of a cash bond without any risk, the seller must purchase the bond and hold it until delivery. This involves the short-term cost of holding and financing the bonds until delivery and the long-term yield received from the bonds. Therefore, prior to delivery, the futures equivalent price will have to be adjusted for the difference between the interest accrued from the coupon payments and the short-term financing rate (repo rate). The difference is known as the cost of carry and is given by:

$$\textit{Cost of Carry} = \textit{Coupon Income} - \textit{Financing Cost}$$

$$\textit{Cost of Carry} = (A_d + IC - A_s) - \textit{Bond Price} * \textit{Repo Rate} * AF$$

Basis: Although the price of a futures contract will closely track the futures equivalent price of the CTD, a difference will be observed between the two. This difference is known as the basis and is given by:

$$\textit{Basis} = \textit{Futures Equivalent Price} - \textit{Futures Quote}$$

6 Valuation

6.1 Physical Delivery Futures

The valuation of physical delivery futures contract revolves around the CTD bond determination. Once the CTD bond is determined, the price of futures contract is simply

$$Futures\ Equivalent\ Price = \frac{Bond\ Price}{CF}$$

$$Theoretical\ Futures\ Price = Futures\ Equivalent\ Price - \frac{Cost\ of\ Carry}{CF}$$

Bond futures contract has embedded optionality like the delivery option for short, wild card options. FactSet assumes a static CTD when valuing futures that ignore the above optionality. In the near future, we do plan to incorporate the Monte Carlo (MC) model for valuing bond futures of major currencies for which we have the MC model calibrated. However, in scenario analysis or when calculating risk statistics, the static CTD is re-evaluated with the changing rate environment.

6.2 Cash Settled Futures

Cash settled bond futures valuation is straightforward as there is no need to determine CTD bond. These futures all have a synthetic bond with specific characteristics, which serve as nominal CTD. The value of the future is then calculated from a given yield on nominal CTD. The calculation of the yield varies by future contract type.

Australia (AUD): The yield is the average yield of all bonds in the published basket. The futures quote we get is 100 calculated average yield. So, we directly use 100 - quote as the yield to calculate nominal CTD price.

South Korea (KRW): The yield is the average yield of all bonds in the published basket. We calculate the yields of each bond in the basket using quoted prices and set the average value to a nominal CTD price.

Sweden (SEK): The Swedish bond futures quote is the yield that is directly used to value nominal CTD.

Once the yield is determined, nominal CTD has a fixed number of coupons to maturity and the price is calculated as below,

$$P = N * \left(\frac{c}{f} * \frac{1-(1+\frac{y}{f})^{-T*f}}{\frac{y}{f}} + (1+\frac{y}{f})^{-T*f} \right)$$

$$P = Futures\ Price$$

$$N = Nominal\ contract\ amount$$

$$c = Factor\ Rate$$

$$f = Payment\ frequency\ of\ nominal\ bonde$$

$$y = Yield\ calculated\ as\ above$$

$$T = Tenor\ of\ the\ nominal\ bond\ per\ future\ type$$

7 Options on Bond Futures

In general, options on money market interest rate are American style and the pay-off for a call at the expiration date is described as:

$$V(T) = max(P(T) - K, 0) = max((100 - f(T)) - (100 - K)) = max(K - f(T))$$
$$V(t) = e{-}r(T{-}t)EB[V(T)|t)$$

Where P is the future price, K is the strike price, the payoff function V is convex, and e−r(T−t) is the discount factor.

Most of the options traded on the exchange have a daily margin process. From formula (3) and Jensen's inequality, we can derive:

$$EB[V(T)|t) > max(EB(P(T)|t) - K) = max(P(t) - K) \quad (1)$$

This shows it would never be optimal to exercise the American option. FactSet values all money market options as **European** style.

From (3), we also know that future price P(t) and derived rate f(t) is a martingale under spot measure, in which case f(t) can be modeled as:

$$df = \sigma fkdW(t) \quad (2)$$

FactSet provides analytics deriving from two common models:

Black Model: when k = 1, the futures price follows a log-normal Brownian motion with constant volatility σ

$$C(t) = e^{-r(T-t)}(f(t)N(d1) - KN(d2))$$
$$P(t) = e^{-r(T-t)}(-f(t)N(-d1) + KN(-d2))$$
$$d1 = \frac{ln(f(t)/K) + \sigma^2(T-t)/2}{\sigma\sqrt{T-t}}$$
$$d2 = d1 - \sigma\sqrt{T-t}$$

Given the put-call parity relationship, the below holds for FactSet-derived option analytics:

$$DeltaCall = DeltaPut + e{-}r(T{-}t)$$
$$GammaCall = GammaPut$$
$$ThetaCall + r * K * e{-}r(T{-}t) = ThetaPut + r * f(t) * e{-}r(T{-}t)$$
$$RhoCall - (T - t) * K * e{-}r(T{-}t) = RhoPut - (T - t) * f(t) * e{-}r(T{-}t)$$
$$VegaCall = VegaPut$$

Bachelier Model: when k = 0, the futures price follows normal Brownian motion with constant volatility σ:

$$C(t) = e^{-r(T-t)}((f(t) - K)N(d) + \sigma\sqrt{T-t}\phi(d)))$$
$$P(t) = e^{-r(T-t)}((-f(t) + K)N(-d) + \sigma\sqrt{T-t}\phi(d)))$$
$$d = \frac{f(t) - K}{\sigma\sqrt{T-t}}$$

8 Bond Futures Return Attribution

8.1 Physical Delivery Futures

The futures price will be calculated by repricing the underlying CTD bond, recalculating carry, and financing cost under different attribution assumptions.

Starting/Ending Price

$$Starting/EndingPrice = \begin{cases} InputtedPrice & Attributionmethod = InputtedPrice \\ ExchangeQuotedPrice & Attributionmethod = OAS \end{cases}$$

If the attribution method is input price, then we reconcile the futures market quote with theoretical futures price using a plugin Net Basis Spread component. This spread is kept constant and added over calculated futures price in each attribution component.

Net Basis Spread = Starting Price - Future Theoretical Price at Start.

Carry Effect: There will be no need to calculate carry effect since the future theoretical value calculation in following price components would automatically take care of it correctly by having both coupon effect and financing cost.

Once the basis spread is established, the return calculations follow standard FactSet RA methodology Price Return Components

$$AccretionPrice = StartingPrice = FutureTheoreticalPrice(Tstart, Curvestart, VolStart) + NetBasisSpread \quad (3)$$

$$RolldownPrice = FutureTheoreticalPrice(Tend, CurveStart, VolStart) + NetBasisSpread \quad (4)$$

$$ShiftPrice = FutureTheoreticalPrice(Tend, Curveshift, VolStart) + NetBasisSpread \quad (5)$$

$$TwistPrice = FutureTheoreticalPrice(Tend, Curvetwist, VolStart) + NetBasisSpread \quad (6)$$

$$ShapePrice = FutureTheoreticalPrice(Tend, Curveend, VolStart) + NetBasisSpread \quad (7)$$

$$VolPrice = FutureTheoreticalPrice(Tend, Curveend, Volend) + NetBasisSpread \quad (8)$$

8.2 Cash Settled Futures

Cash settle bond futures such as in Australia, Korea, and Sweden all have a determined formula to calculate the future value, which is also used in mark-to-market calculations.

The calculation involves a fixed-maturity synthetic bond as underlying, which hints no carry return should be reported. Since the cash-settle bond future does not have CTD options and the pricing would be based on the exchange published formula, no volatility return would be reported. So, the special handling for cash settled futures is

$$RolldownPrice = AccretionPrice = StartingPrice$$

$$VolPrice = ShapePrice$$

The Net Basis would be needed for Korean futures since it is quoted as Price. For Australian and Swedish futures, it would be 0 since the quote is on yield and the price would be directly calculated from the quote.

9 Appendix

References

1. John C. Hull Options, Futures, and Other Derivatives (9th Edition). ISBN-13: 978-0133456318
2. Montreal Exchange, Bond Futures Reference Manual.
   https://www.m-x.ca/f_publications_en/bond_futures_manual_en.pdf
3. CME, Calculating U.S. Treasury Futures Conversion Factor.
   https://www.cmegroup.com/trading/interest-rates/files/Calculating_U.S.Treasury_Futures_Conversion_Factors.pdf

ABOUT FACTSET

FactSet (NYSE:FDS | NASDAQ:FDS) delivers superior content, analytics, and flexible technology to help more than 160,000 users see and seize opportunity sooner. We give investment professionals the edge to outperform with informed insights, workflow solutions across the portfolio lifecycle, and industry-leading support from dedicated specialists. We're proud to have been recognized with multiple awards for our analytical and data-driven solutions and repeatedly scored 100 by the Human Rights Campaign® Corporate Equality Index for our LGBTQ+ inclusive policies and practices. Subscribe to our thought leadership blog to get fresh insight delivered daily at insight.factset.com.

Learn more at www.factset.com and follow us on Twitter: www.twitter.com/factset.

IMPORTANT NOTICE