

SWAP CURVE BUILDING AT FACTSET: THE MULTI-CURVE FRAMEWORK

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1 Introduction

The interest rate swap (IRS) market is the third largest market in the U.S. for interest rate securities after U.S. Treasuries and mortgage backed securities (MBS), as demonstrated by Table 1. Interest rate swap curves are important not just for valuing swaps, but also for their role in determining the market expectation of future LIBOR fixings, since many financial securities have coupons that are set based on this fixing. Financial markets changed significantly due to the global financial crisis (GFC) of 2008, none more so than the IRS market. The market changes were so disruptive that they caused a reexamination of the entire foundation of quantitative finance.

	Security Type	Gross Market Value
	U.S. Treasuries	17.57 Trillion USD
	U.S. Mortgage Backed Securities	10.076 Trillion USD
	Interest Rate Derivatives	1.434 Trillion USD

Table 1: U.S. market sizes as of Q4 2017. The U.S. Treasury and the Interest Rate Derivatives data are taken from the Bank of International Settlements, and the U.S. mortgage data is taken from the Federal Reserve Bank of St. Louis. The IRD market may look small in comparison; however, the notional amount outstanding is 156.5 trillion USD.

To begin to understand why these changes were so disruptive, a basic refresher on IRS is useful. A swap is a legal obligation between two counterparties to exchange one set of future cash flows for another, and can come in many forms. This paper focuses on two specific types: vanilla IRS, where one party pays a fixed coupon (the fixed leg) and the other party pays a floating coupon indexed to a specified observable interest rate in the same currency (the floating leg), and basis swaps, where both parties pay floating coupons indexed to different interest rates in the same currency. The magnitude of the cash flows depends on a notional principal value (simply referred to as “the notional”). This value plays the same role as a bond principal value, the difference being that the notional is never exchanged as there would be no economic impact. The nomenclature of an IRS is from the point of view of the fixed leg; a payer swap is a swap where the counterparty is long the floating leg and short the fixed leg, whereas a receiver swap is a swap where the counterparty is long the fixed leg and short the floating leg. Further, the notion of “long” and “short” a swap also refers to the fixed leg: the counterparty is long the swap if they receive the fixed leg.

2 Floating Leg Indices

The interest rate index used in the majority of swaps is a trimmed average of the results of a poll, where an administering body polls a set number of contributing banks for what they could borrow unsecured for a specified amount of time, known as a tenor, such as three or six months. For example, the Intercontinental Exchange (ICE) queries between 11 and 17 major banks in London to produce daily London Interbank Offered Rate (LIBOR) quotations for five currencies (CHF, EUR, GBP, JPY, USD) and seven tenors (S/N, 1w, 1m, 2m, 3m, 6m, 12m). Other governing bodies produce indices in a similar manner; for example, the European Money Market Institute (EMMI) polls 20 contributing banks in the Eurozone in order to determine the daily quotations of the EURO Interbank Offered Rate (EURIBOR).

There are economies where the British Bankers Association (BBA, who administered LIBOR prior to ICE) discontinued quoting LIBOR, including Denmark, Sweden, Canada, Australia, and New Zealand. Each country had to decide how it would set the floating leg of an IRS in their currency. Most countries decided to follow the same mechanism by appointing an administrator and conducting polls of major institutions. However, Australia and New Zealand decided to take a different course and opted not for an index set by a poll, but rather by averaging quotes of actual market traded securities – the Australian Bank Bill Swap Rate (BBSW) and the New Zealand Bank Bill Rate (BKBM), respectively.

3 Swap Pricing

Prior to the GFC, a single yield curve was the output of the process of curve construction. Many believed that this concept was a result of the unassailable law of no-arbitrage, since a replicating portfolio can be set up that uniquely determines the implied rate between two times. To see this, begin with the discount curve, which is the curve whose value at time t is the value of a zero coupon bond maturing at that time; in other words the value today of one dollar to be paid at time t . Suppose one party agrees to receive one dollar at time t_1 and repay it at a later time $t_1 + \tau$, then the implied (simply compounded) rate between the two times can be nothing other than

$$L(t; t_1, t_1 + \tau) = \frac{1}{\tau} \left(\frac{D(t_1)}{D(t_1 + \tau)} - 1 \right) \tag{1}$$

otherwise one party can enter the agreement and purchase zero coupon bonds in the market and earn a guaranteed profit. Note that there are three time points in the definition of a LIBOR rate:

1. t – time at which the rate is observed
2. t_1 – start of the accrual period, where the rate is in effect
3. τ – length of the accrual period, equivalent to the tenor defined previously

To ease notation, the shorthand $L_\tau(t; T)$ is used, defined as the LIBOR rate observed at time t for borrowing between T and $T + \tau$. For example, in this notation the LIBOR rate as seen today for lending in three months time for a term of three months is written $L_{3m}(0; 0.25)$. The initial observation time will not be important for curve construction, but will become very important when considering models of the evolution of the term structure of interest rates.

Equation 1 implicitly contains the usual compounding relationship, where the appropriate average of shorter term rates also uniquely determines the value of longer-term rates

$$(1 + \tau L_{3m}(t; t))(1 + \tau L_{3m}(t; t + 3m)) = (1 + \tau L_{6m}(t; t)). \tag{2}$$

Intuitively, this result is nothing more than asserting that credit risk is not present in the interbank market – all participants have equal utility between borrowing for two three-month periods and borrowing for a single six-month period. Equation 1 provides LIBOR rate projections, and hence the curve $L_\tau(t; T)$ is known as the projection curve.

The product of the coupon, the accrual fraction, and the NPV determines each individual cash flow. The pre-crisis price of an interest rate swap is the sum of the present value of all incoming (receive leg) cash flows, less the sum of the present values of all of the outgoing (pay leg) cash flows, with discounting calculated using the same discount curve that the floating leg coupon is projected via Equation 1

$$S = \sum_{i=1}^n D(t, t_i) N c \tau_i - \sum_{j=1}^n D(t, t_j) N L_\tau(t; t_{j-1}) \tau_j, \tag{3}$$

where N is the notional, $D(t, T)$ is the price of a zero-coupon observed at time t maturing at time T (the discount factor), and τ is the year fraction between successive payment dates $\{t_i\}$.

There are a number of things of note in Equation 3. First, the fixed leg and floating leg can have a different frequency, and therefore both require different summation indices. In fact, the market standard for both the USD LIBOR and EURIBOR swaps is different payment frequencies on the fixed and float legs. Second, the magnitude of the cash flow changes based on the accrual fraction of each specific coupon period, in opposition to the behavior of bonds that pay a constant fixed amount on each payment date. Third, the curve used to discount future cash flows $D(t, T)$ is known as the discount curve, which, in the single curve paradigm, is the curve used when determining the projection curve.

The market quotes for vanilla IRS provided from market data vendors are par swap rates, which are defined as the set of unique values of the coupon c in Equation 3 that makes both legs have equal value as a function of tenor t_n , and therefore the present value of the swap is zero. The par swap rate is a weighted average of future LIBOR projections, determined by setting $S = 0$ in Equation 3

$$c_{\text{par}} = \sum_i \omega_i L_\tau(t; t_{i-1}), \tag{4}$$

where

$$\omega_i = \frac{D(t_i) \tau_i}{\sum_j D(t_j) L_\tau(t; t_{j-1}) \tau_j}. \tag{5}$$

Pricing a basis swap follows the same, replacing the fixed leg in Equation 1 with a second floating leg with a different tenor

$$S = \sum_{i=1}^N D(t_i) N (L_{\tau_1}(t; t_{i-1}) + \delta) \tau_i - \sum_{j=1}^N N L_{\tau_2}(t; t_{j-1}) \tau_j. \tag{6}$$

A spread δ is added to the leg with the shorter-tenor (in this case the receiving leg). The market data quotes for basis swaps are the basis spread δ that produces an equal value on each leg, resulting in a par (zero) value for the swap. Note that in the single curve paradigm, this spread should be identically zero and prior to the GFC the market quotes were very small, as evidenced by Figure 1.

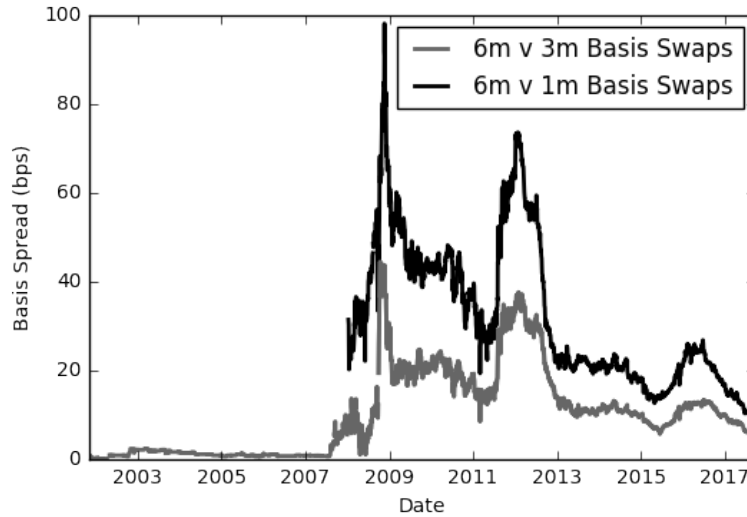


Figure 1: Basis swap market data in the Euro market historically from 2003 through 2017. Note the nearly constant basis spreads prior to 2008, representing only the spread retained by the investment bank, and the persistent non-zero spreads afterward, representing a non-zero credit component of the interbank market.

4 Impact of the Financial Crisis on the Swap Market

The default of Lehman Brothers showed that it is impossible to ignore credit risk in the interbank market. Consequently, the implicit assumption that a twice-compounded three-month LIBOR is economically equivalent to a six-month LIBOR is no longer valid – in other words, Equation 1 no longer holds. If a borrower can default, the lender must add a spread to the lending rate for longer-tenor loans to compensate for possible losses. The signal that proved this came from the basis swap market. Figure 1 shows the data for swapping different tenors of EURIBOR for six-month EURIBOR. Around 2008 the basis spreads spiked, going from near zero to hundreds of basis points. Recall that this is the spread on the shorter-tenor floating leg required to enter this swap at par (zero) value. Therefore, the interbank market signaled that the projections of longer-term LIBOR based on compounding alone were too low, as they now believed a credit component will be present in the future LIBOR fixings. The impacts of this are pervasive. First, each LIBOR tenor now needs its own projection curve

$$L_{\tau}(t; t_1, t_1 + \tau) = \frac{1}{\tau} \left(\frac{D_{\tau}(t_1)}{D_{\tau}(t_1 + \tau)} - 1 \right). \tag{7}$$

Second, and more importantly, given that there is now a family of curves, it is not clear which of these underlying discount curves $D_{\tau}(t)$ to use for discounting future cash flows. Two answers were proposed – a theoretical answer and a practical answer.

5 Collateral Discounting and Overnight Index Swaps

The LIBOR rate now contains a credit risk component and therefore it is no longer a valid proxy for the risk-free rate – the rate singled out in arbitrage-free pricing theory as the rate to use for discounting. The theoretical answer was put forward by John Hull and Alan White [1]:

In our view the best proxy available for the risk-free rate is the OIS (overnight index swap) rate. Many dealers now use the OIS rate instead when valuing derivatives backed by collateral. Eventually, they may do the same for all portfolios.

The practical answer, already hinted at in the quote by Hull and White, was that the overnight rate should be used for discounting, as this is the rate that applies to collateral as specified in the credit support annex (a legal addendum to all OTC derivative contracts).

In an overnight index swap, one counterparty pays a fixed rate and receives the daily-compounded overnight rate in the economy. For instance, in the U.S., this rate is the Fed Funds rate, and in Great Britain, it is the Sterling Overnight Index Average (SONIA).

On the other hand, swap traders argue that the OIS is the correct rate, but for a different reason. To understand their reasoning, consider a fully collateralized swap that is marked to market daily with one remaining cash flow. The mechanics of collateral accounts dictate that the counterparty for whom the swap is a liability has to pay into the collateral account and pay interest at the overnight rate. This counterparty wants to reduce capital tied up in collateral and therefore will put in enough money today that, given that interest is accruing daily at the OIS rate, will exactly equal the obligation on the future cash flow date. Put in other words, the amount today of the liability is the future value discounted by the overnight rate.

This link between discounting and funding is controversial – however, what is not controversial is that the swap market now embraces overnight discounting. Today the London Clearing House (LCH), the Chicago Mercantile Exchange (CME), and major accounting firms all discount swaps using the overnight rate for determining their present value. Furthermore, the interdealer broker that supplies FactSet with par swap quotes also indicated that their quotes assume overnight discounting. The most compelling evidence came from a numerical study of forward starting interest rate swaps done by Bianchetti and Carlicchi [2] that shows the current market practice to price securities quoted in the swap market is to use the OIS curve.

6 Swap Curve Building

The ultimate goal of modern swap curve building is to produce a family of projection curves for commonly traded tenors (1m, 3m, and 6m) together with the OIS discounting curve. Most swap markets are homogeneous in that they have a single standard tenor quoted for the entire set of data. This standard tenor is typically 6m, except for the United States, Hong Kong, New Zealand, and South Africa, which use 3m. The only economy that is not homogeneous is Australia, whose market quotes represent 3m swaps up to three years and 6m afterward. The method of construction presented in this chapter will not cover this case; however, the spirit remains the same.

6.1 Instrument Selection

The mathematics of swap curve construction is not difficult – indeed almost all of the required mathematics is contained within Equations 1 and 3 – yet curve construction is nuanced and must be done with extreme care. Accurate security pricing is at the core of swap curve building. Market data represents more than a number, it represents a tradable security that, if transacted, can alter the characteristics of a portfolio. Instead of simply connecting the dots between market data points, the process of bootstrapping treats each market data quote as a bona fide security, together with all of its daycounts and conventions. Before describing the details of bootstrapping, the next section will focus on the selection of the securities on which to base the swap curve.

1. Cash Deposits (CDs) are money market instruments that pay a fixed amount at maturities ranging from days to weeks. The market data is quoted in annualized simple rate, meaning that if the rate is R then the amount received at time $t + \tau$ is $N(1 + R\tau)$, where the initial cost at time t is N .
2. Forward Rate Agreements (FRAs) are securities that fix a rate of borrowing between two times in the future, with maturities from months to years. The convention is to quote them as $M \times N$, meaning the borrowing period begins in M months and lasts for $M - N$ months.
3. Eurodollar Future (EDF) contracts are liquid exchange traded contracts for borrowing between two times. These are very similar to FRAs, the difference being that EDFs are exchange traded and marked to market daily. Another difference is that the maturities are on fixed dates (known as International Money Market IMM dates) as opposed to relative dates, and as the maturity approaches, new longer contracts are issued.
4. Vanilla IRS with maturities from one to 50 years.
5. Overnight swaps with maturities from one week to 50 years.
6. Basis swaps, quoted in basis points with maturities ranging from one to 50 years.

If a particular security is available, it is not necessarily well suited for inclusion in the curve construction process. The securities chosen should move coherently as market sentiment/expectations change; therefore, those securities with sufficient liquidity are candidates.

Liquidity is a crucial characteristic when determining the subset of all available securities to use in curve construction. If a security lacks liquidity, then the associated data point may be “stale” in other words, the quote did not update coherently with the rest of the securities. For example, the 6m FRA indexed to the GBP LIBOR 6m swap did not move coherently with the rest of the GBP LIBOR 6m swaps for two days after the Brexit vote, producing a nonsensical curve. Removing this point for the week following Brexit returned the curve to a better shape. This demonstrates the importance of being able to determine dynamically the set of market securities to incorporate into curve construction.

6.2 Bootstrapping

Construction of the swap curves occurs sequentially, beginning with the OIS curve, by a process known as bootstrapping. The main idea of bootstrapping is to extend a curve iteratively by ensuring that the price of each successive security is in perfect agreement with the market data; for instance, if the bootstrapping instrument is a swap, the curve is constructed such that the price of the swap is identically zero. This is in contrast to sovereign bond yield curve construction, which typically proceeds by fitting a parametric form of a curve to very carefully selected bonds. The reason for the difference is that, unlike swaps, bonds are not issued on a daily basis. Off-the-run

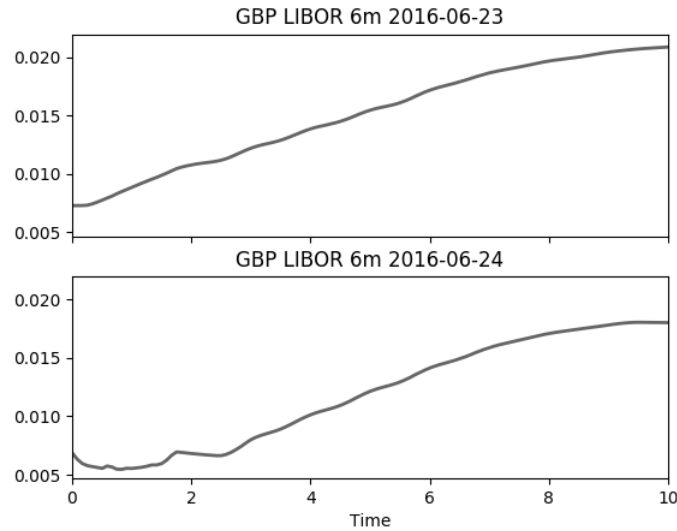


Figure 2: Kink in the 2y point seen in the GBP LIBOR 6m curve the week following the Brexit vote on March 29, 2017.

bonds are more thinly traded and are therefore not in equilibrium due to the lack of demand, whereas the first on-the-run bonds can be out of equilibrium due to a surge in demand. The bonds chosen should be in equilibrium of supply and demand as much as possible. Since this will only encompass a small number of bonds, iterative bootstrapping is impossible. Many central banks around the world use the Nelson-Siegel [3] or the Svensson [4] parameterization, both of which are econometrically motivated to be able to produce many, if not all, empirically observed yield curve shapes ¹.

Since swaps are originated daily, interdealer brokers can monitor indicative par swap quotes on an intraday frequency. These quotes run from years to decades, and therefore there is plenty of data in equilibrium, and a curve shape needn't be mandated *a priori*. The curve shape is an output of bootstrapping, predominantly affected by the choice of interpolation method. Many interpolation methods were developed in engineering and scientific disciplines and therefore did not take into account financial considerations. Important properties of interpolation methods are smoothness and locality, which are actually competing properties.

Smoothness of the implied instantaneous forward rate curve is important, since discontinuous² forward rates imply “either implausible expectations about future short-term interest rates, or implausible expectations about holding period returns.”[6]

Locality, on the other hand, is the property of the curve that, when a single input changes, the effect on the curve is localized around the tenor of the change. For instance, if the 5y swap increases, leaving all other inputs the same, then the resulting curve should be very close to the original curve, except for the curve region near the 5y point. To understand why this is important, consider key rate duration (or bucket DV01) hedging.

¹For a description of FactSet’s sovereign curve building, please refer to the whitepaper by Mossessian and Davis [5].

²Non C_0 curves.



Figure 3: Repricing swaps of various tenors off of the constructed EURIBOR 6m curve from the beginning of 2017 to September 16, 2018. The series represents the bid, mid, and ask rates normalized by the derived par swap rate. The “mid” series should be approximately zero for a well-behaved curve.

Repricing tests guarantee that the curve is consistent with the raw data inputs. Explicitly, the par swap rates derived from the resulting curve are exactly equivalent to the market quoted par swap rates used during curve bootstrapping. This is a minimal test, and ensures that the swap curve embodies the market expectation of future LIBOR fixings. The excluded instruments, which represent lower liquidity, are also repriced; however, the curve will not price these swaps to par. Therefore, the difference between the market par swap rate and the derived par swap rate is an important indicator of whether the curve is reasonable, even in the regions where no *a priori* information was provided. There is a caveat with this test, however. Outliers do not immediately prove that the curve is not high quality, as the specific market data point may be itself the outlier. Figure 3 displays the results of repricing excluded instruments on the EURIBOR 6m curve from the beginning of 2017 to September 2018.

7 Conclusion

The ultimate goal of swap curve building is a set of curves for each economy including the overnight curve and the projection curve for tenors that have an active swap, or basis swap, market. For example, Figure 4 shows the EONIA

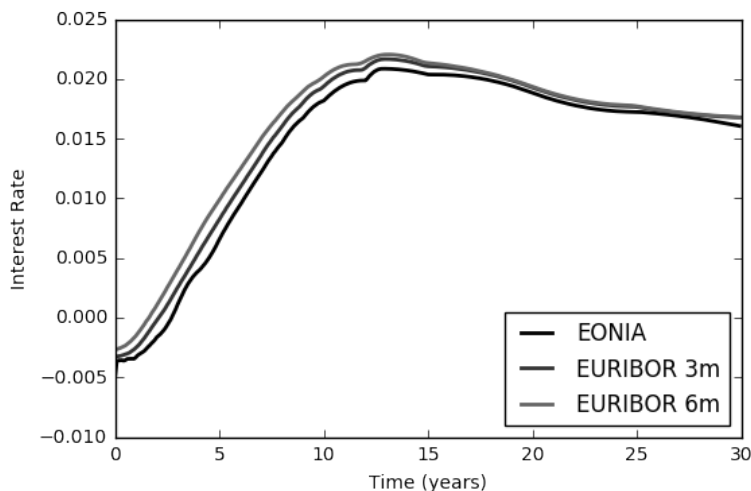


Figure 4: Swap curves for the Euro zone as of September 20, 2017.

(European overnight index average) curve together with the EURIBOR 3m and 6m curves for September 20, 2017. Although not mathematically challenging, swap curve building is nuanced. Every market data point cannot be treated simply as a point to interpolate through, but as a real financial security, with all of its conventions.

Interest rate swap curves are of central importance in valuing a very large portion of debt markets, from floating rate bonds, to mortgages, to interest rate derivatives. Prior to the global financial crisis, one single curve was sufficient to determine expected LIBOR rates of all tenors. However, when Lehman defaulted, credit was no longer negligible in interbank markets. Therefore, banks would command a premium for longer-term borrowing, and hence a separate curve is now necessary for each LIBOR tenor. These curves incorporate the credit risk inherent in the interbank market, something that was assumed to be negligible prior to 2008.

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